AN OPTIMUM RADIATOR-FIN PROFILE

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A variational problem on the determination of an optimum-weight radiator-fin profile is formulated so as to result in a uniquely defined solution. The results of the computer-derived solution are presented here. The effect of certain structural factors on the weight of the radiator fin is analyzed.

Many papers have been devoted to the study of radiator fins, and these deal particularly with a variety

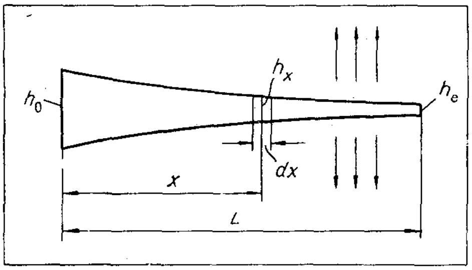


Fig. 1. Design diagram.

of profiles, including: flat [1–4], triangular [3, 5], trapezoidal [3], and those with a constant temperature gradient [3]. It is demonstrated that the above-enumerated complex-profile fins are lighter in weight, all other conditions being equal, than the flat fin. However, these are not the lightest fins. Reference [6] is devoted to the solution of the variational problem concerned with finding the optimum-weight radiator-fin profile. The author of that reference, desirous of achieving an analytical solution, used the function of the change in temperature over the fin as the unknown extremal and introduced the ratio of the fin-tip temperature to the fin-base temperature as the specified initial quantity. This prevented the unique definition of the derived solution. However, from the physical standpoint the requirement of achieving a fin of the lowest possible weight must also specify uniquely the optimum profile of the fin and its dimensions.

Below we present that formulation of the variational problem concerned with determining the optimum-weight radiator-fin profile which leads to a uniquely defined solution, and we also present that solution.

Let us transform to dimensionless form the heat-balance equation written for a radiator-fin element (Fig. 1), with the standard assumptions [3],

$$\frac{d(\lambda h_x dT/dx)}{dt} = 2\varepsilon\sigma T^4,$$

using the following dimensionless quantities:

$$\varphi = h_x/h_0,\ \theta = T/T_0,\ t = x/L,\ c = 2\varepsilon\sigma T_0^3 L^2/\lambda h_0. \quad (1)$$

Here T is the fin temperature at cross section x; T_0 is the fin-base temperature; λ is the thermal conductivity of the fin material; ε is the surface emissivity of the fin; σ is the Stefan-Boltzmann constant.

After transformation, we derive the equation

$$\frac{d(\varphi\, d\theta/dt)}{dt} = c\,\theta^4 \quad (2)$$

with the boundary conditions

$$1)\ \theta = 1 \text{ when } t = 0,\ 2)\ d\theta/dt = 0 \text{ when } t = 1.$$

Let us write the expression for the specific weight γ of the fin:

$$\gamma = \frac{G}{Q_{rem}} = \frac{g\rho h_0 \int_0^1 \varphi\, dt}{Q_{rem}},$$

where G is the weight of the fin; ρ is the density of fin material; Q_{rem} is the heat removed to the surrounding space per unit of fin width, and this, in turn, can be expressed as

$$Q_{rem} = 2\varepsilon\sigma T_0^4 L \int_0^1 \theta^4\, dt. \quad (3)$$

Using (1) and (3), by carrying out certain substitutions we obtain

$$\gamma = g\frac{\rho}{\lambda}\frac{Q_{rem}^2}{4\varepsilon^2\sigma^2 T_0^9}\frac{\int_0^1 \varphi\, dt}{c\left(\int_0^1 \theta^4\, dt\right)^3}.$$

We see from this expression that the fin of the lowest weight corresponds to the minimum of the functional

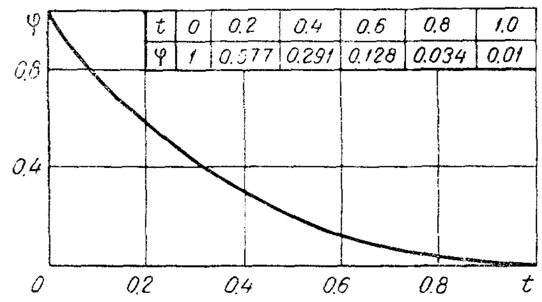
$$\Phi = \frac{\int_0^1 \varphi\, dt}{c\left(\int_0^1 \theta^4\, dt\right)^3},$$

which is a function of the dimensionless fin-conductivity parameter c and of the fin profile, expressed in dimensionless form by the function $\varphi(t)$, which is the unknown extremal.

The dimensions of the radiator fin are determined from the following expressions:

$$L = Q_{rem}/2\varepsilon\sigma T_0^4 \int_0^1 \theta^4\, dt,$$

$$h_0 = 2Q_{rem}^2/\lambda\varepsilon\sigma T_0^5 c\left(\int_0^1 \theta^4\, dt\right)^2.$$



t	0	0.2	0.4	0.6	0.8	1.0
φ	1	0.577	0.291	0.128	0.034	0.01

Fig. 2. Profile of optimum fin.

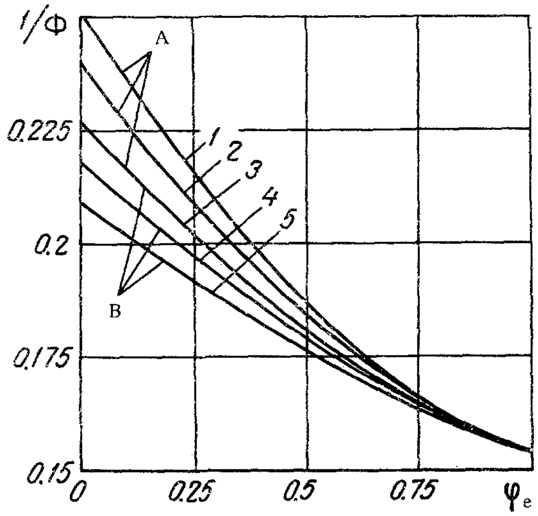


Fig. 3. Fin weight versus relative thickness at the end of fin.

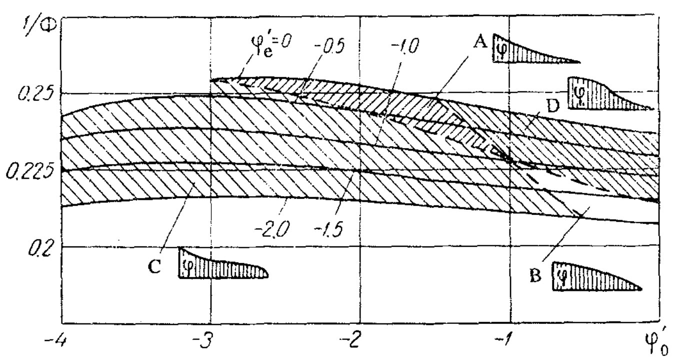


Fig. 4. Fin weight versus value of derivative φ' at fin base and at its end: A) region of concave profiles; B) region of convex profiles; C) region of concave-convex profiles; D) region of convex-concave profiles; cross-point of curves—triangular fin.

It is impossible to find the analytical extremal $\varphi(t)$ when the relationship between the functions φ, θ and the parameter c is specified by the second-order nonlinear differential equation (2) with two-point boundary conditions. For this purpose we will use one of the direct methods for solving variational problems—the method of functional descent [7, 8], which reduces the solution of the variational problem to the solution of the problem concerned with finding the optimum of the function for many variables. This method presupposes the preliminary specification of the form of the function being sought. In our case we know that the unknown function must be positive and monotonically diminishing. It is only at the end of the interval, when t = 1, that the function φ can vanish. In particular, these requirements are satisfied by a third-degree polynomial. The coefficients of the polynomial are determined in this case from the values of the function φ and its derivative φ' at the base of the fin (φ_0, φ_0') and at the end of the fin (φ_e, φ_e'):

$$\varphi = 1 + at + bt^2 + dt^3, \quad (4)$$

where

$$a = \varphi_0',$$

$$b = 3\varphi_e - \varphi_e' - 3 - 2\varphi_0',$$

$$d = 2\varphi_0' - 2\varphi_e + \varphi_e'.$$

The functional Φ thus becomes a function of four independent variables c, φ_0', φ_e, and φ_e'.

The minimum of Φ was determined with the aid of the digital M-20 computer. Given fixed values for c, φ_0', φ_e, and φ_e', for the solution of Eq. (2), the latter was presented in the following finite-difference form:

$$\theta_{i+1} = \theta_i + (d\theta/dt)_i \Delta t,$$

$$(d\theta/dt)_{i+1} = \left[c\,\theta_i^4 \Delta t - (d\theta/dt)_i \varphi_i\right]/\varphi_{i+1}.$$

The boundary condition $d\theta/dt = 0$ when t = 1 was satisfied by proper selection of $(d\theta/dt)_0$.

We see from the above-cited formulas that the computer solution assumes that the function φ is not equal to zero, even at the tip of the fin. In this connection, the minimum assumed value of φ was not zero, but 0.01. In practical terms, this distinction has no effect on the optimum profile, nor on the dimensions of the fin.

Optimization resulted in a minimum value $\Phi = 3.93$, which corresponds to c = 0.9, $\varphi_0' = -2.5$, $\varphi_0' = 0$, and $\varphi_e = 0.01$. The magnitude of the integral $\int_0^1 \theta^4 dt$, normally referred to as the efficiency of the fin, and determining the length of the fin, is equal to 0.438. The optimum-fin profile in accordance with formulas (4) is determined by the equation

$$\varphi = 1 - 2.5t + 2.03t^2 - 0.52t^3$$

and is shown in Fig. 2.

In order to produce fins that are easier to fabricate, it is very important that we know the influence exerted by the parameters which govern the fin profile on the weight of the fin and on its dimensions. The proposed method of solution permits us to undertake such an analysis. It develops that the conductivity parameter c has but a slight effect in the 0.8–1.0 interval on the weight of the optimum fin. The results from an evaluation of the effect exerted by φ_e on the weight of the fin for certain specific functions φ are shown in Fig. 3. Curves A correspond to concave fins

$$\varphi = 1 + (\varphi_e - 1)(2 - u)t + (u - 1)(\varphi_e - 1)t^2,$$

while curves B correspond to convex fins

$$\varphi = 1 + u(\varphi_e - 1)t + (1 - u)(\varphi_e - 1)t^2.$$

Figure 4 shows the effect of the derivatives φ' at the base of the fin (φ_0') and at the tip of the fin (φ_e') on the weight of the fin when the curve for φ is specified in the form of (4) for $\varphi_e = 0.01$ and c = 0.9.

From the derived data we can draw the conclusion that the values of the function φ and its derivatives at the tip of the fin (φ_e and φ_e') exert particularly significant influence on the weight of the fin; the quantities c and φ_0', however, have very little effect on fin weight.

Comparison of an optimum-profile fin relative to a triangular fin shows that the former is 11% lighter, 15% longer, and 33% thicker at the base.

REFERENCES

1. Y. G. Bartas and W. H. Sellers, Trans. ASME, ser. C, J. of Heat Transfer, 82, no. 1, 1960.

2. M. Sparrow and R. G. Eckert, J. Aero-Space Sciences, October, 28, no. 10, 1961.

3. D. B. McCoy, Design of Space Powerplants [Russian translation], Izd. Mashinostroenie, 1966.

4. Y. L. Chen, J. Aero-Space Sci., 27, no. 11, 1960.

5. E. N. Nilson and R. Curry, IASS, 27, no. 2, 1960.

6. G. L. Grodzovskii, Izv. AN SSSR, Energetika i avtomatika, no. 6, 1962.

7. E. F. Beckenback, Modern Mathematics for the Engineer [Russian translation], IL, 1958.

8. H. J. Kelly, collection: G. Leitman, ed., Optimization Techniques with Applications to Aerospace Systems [Russian translation], Izd. Nauka, 1965.

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